G:5HB" - 326"I562J0"K2>6" G:5HB"L>81>:E62"I>6M260".1>E:" G:5HB"L>81N"O:5>40"PE720.1>E:" # JB"RSJ1>E6"T99>JE" # JB" #>1>"O>92U2:" # JB" P2J6>"O>VE7" G:5HB"T9EW>E3"O>U>6" #JB"KE>6>"O>::2:5" # JB"TJCE8>"O1S8>6E"PEV" G:5HB" # 2W>6"O532:" G:5HB"I266EH2:"O:>68" G:5HB".3>:2"O:288" G:5HB"L>81N"O:5>4" #:B"T>:56"O:5X6" #JB")>C>681>"OS:6J" K:B"T64:2X".>CA9233" G:5HB".>:53".>CA9233" #JB"/>:>".>:8C>32" G:5HB" . 1>:32J" . 126" G:5HB"RS81". 1E34J" G:5HB"T66>"L>8N6" . 1CE232XJUE" #:B"T3H:245".15X" G:5HB"T662" # >:E2" . 1S432EW1" G:5HB"YE53>". 325D5S35S" K:B"P>32:E2"K>C>J75" G:5HB"KE>62"+>:C2:"

*+,-,./0!

JB" # 56EZS2"+3>77>D2685" $G:5HB^{"}RS9[6^{"}->M8>C9E420]$ +2:6\642M" # JB", 54>" - 1>:E9" K:B") 123456" - :>9U2" # JB" (CE3N", 2785:" #:B"ISJ&E6", 5335X>N" # JB", 2326", S>6W" #:B"K2:2U", S68" K:B", E3>:N"\$6X554" # JB"%>4EE>"L>71N6JU>" G:5HB"%5:C>64"/>9:E2" G:5HB"L>6W"/22" G:5HB" (CC>6S2332" / 2"GE71560 P5:J8C>6" #:B"\$>6"/ESVE>"'E>6" #:B"P>:S6" #>3EU" K:B".>:3N" #>6E56" G:5HB"R1564>" #>:8E6SJJ26" K:B"L266281" # 7%2E33N" # JB", 26: V28>" # 272" G:5HB") 1>1:M>4" # 5V>9" G:5HB"K>DE4" # 5682CS::5" G:5HB"R5N" # 55432N" #:B"/2AE6W" #5S" G:5HB" # E71232"G282:J560O>4>3E" G:5HB"T6W23>"GN32"

#JB"I2>6>662"R592:8J56" G:5⊞".:2J5")\" #:B"RN>6")1558" #:B")>E4")E4>6E" K:B"I2JJ2")ECJ" # JB"I>6E72") A2672:" G:5HB") 12332N") 8>WW"G282:J56" G:5HB"/>6>")82:C>7" #:B" #>88") 85453>U" G:5HB"P>66E6>")M8>E695U" # JB"%5:E6" '>V" # JB"T8126>" '>JJEJ" G:5HB" # EW326>" ' 545:5D>" G:5HB"%V5UE"]>62" #JB", 2>812:"]>88J" # JB"I26>N>"] 299" # JB", >66>"] E7UJ8:5C" K:B"/2J32N"] E3856" G:5HB" (>:3"] 554:SHH"

#\$%&'()"*+"',("#(('\$%-"*+"*\$)(".*&%.\$/0"1234"56"*78592:";<0";=;=">8"?0=="ABCB"DE>"F55C"

"#\$%!&" ' (&#)!

1

89 :1;.<=>,?7,@,./!<A!B+2?3/3<.2>!)2.?!

G:5H2JJ5:"L>81N"O:5>40"C2C92:"5H"812"(^27S8ED2".5CCE88220"75CC26724"71>E:E6W"812"C228E6W0">64" X2375C24"C2C92:J">64"WS2J8"85"812"HE:J8"C228E6W"5H"*\$)(".5S67E3"E6";=;=Q;=;!B"

'12"3>64">7U65X324W2C268"J8>82C268"42D235A24"9N" * \$) (_J"K22A26E6W"L65X324W2"G:5V278"X>J":2>4B""

C9 "D, .3.7!+, @2+;-!

G:E5:"85"A:57224E6W"XE81"812">W264>0"G:5H2JJ5:"O:5>4"A:5DE424">6"5D2:DE2X"5H"812"G>:8E7EA>8E56" - SE423E62J0"X1E71"X2:2"4EJ8:E9S824"E6">4D>672"5H"812"C228E6W"1EW13EW18E6W"JA2>UE6W">64"D58E6W" A:5724S:2J"H5:"812"563E62"H5:C>88"

E9 6, F_3 , =!2. ?!: DD+ $\langle F_2 \rangle$! <A!: 7, . ?2!

'12"TW264>"H5:"812"C228E6W"X>J">AA:5D24B"

G9 6, F3, =!2.?!:DD+<F2>!<A!/H, !I3.5/, -!<A!/H, !*+, F3<5-!I, ./3.7!

E6S82J"5H"812"TA:E3"!`0";=;="C228E6W"5H"*\$)(".5S67E3"X2:2">AA:5D24B""

J9 BH,!%K,15/3F,!&<@@3//,,!L!&2>>!A<+!(<@3.2/3<.-!A+<@!/H,!M><<+!

G:5H2JJ5:"O:5>4"65824"81>80"2>71"N2>:0"62X"C2C92:J"5H"812" (^27S8ED2".5CCE8822">:2"2327824">C56W" .5S67E3"C2C92:J">8"812"HE:J8"C228E6W"5H" * \$) (".5S67E3B") 12"8126"7>3324"H5:"65CE6>8E56J"H:5C"812"H355:"H5:" 812"A5JE8E56"5H".1>E:"5H".5S67E3B" * 6">"C58E56"4S3N"C>420"J2756424">64"7>::E24@"

Q G:5H2JJ5:"L>81>:E62"I>6M26"X>J"2327824">J".1>E:"5H".5S67E3"H5:">"8X5QN2>:"82:CB""

G:5H2JJ5:"O:5>4"756W:>8S3>824"G:5H2JJ5:"I>6M26"56"92E6W"2327824"85"J2:D2"E6"81EJ"ECA5:8>68":532">64" E6DE824"12:"85"71>E:"812":2C>E642:"5H"812"C228E6WB""

G:5H2JJ5:"I>6M26"81>6U24"C2C92:J"5H".5S67E3"H5:"812"D582"5H"756HE42672">64"J8>824"81>8"J12"EJ"355UE6W" H5:X>:4"85"J2:DE6W"812"75CCS6E8N"E6"81EJ"7>A>7E8NB")12"8126"A:5722424"85"7>33"H5:"65CE6>8E56J"H:5C"812" H355:"85"HE33"812":2C>E6E6W"D>7>68"A5JE8E56J"56"812"(^27S8ED2".5CCE8822B"*6"C58E56J"4S3N"C>420"J2756424" >64"7>::E24@"

G:5H2JJ5:"L>81N"O:5>4"X>J"2327824">J"PE72Q.1>E:"5H".5S67E3" G:5H2JJ5:"YE53>".325D5S35S"X>J"2327824">J"812":2A:2J268>8ED2"5H"82>71E6W"J8>HH" K:B"L266281" # 7% 2E33N"X>J"2327824">J"812":2A:2J268>8ED2"5H"5812:">7>42CE7">AA5E6822J" #:B"K2:2U", S68"X>J"2327824">J"812":2A:2J268>8ED2">4CE6EJ8:>8ED2"J8>HH"

N9 O,2.P-!6,D<+/!

 $\label{eq:solution} $$ 12"K2>6"81>6U24"G:5H2JJ5:"I>6M26">64"G:5H2JJ5:"O:5>4"H5:"8>UE6W"56"812":532J"5H". 1>E:">64"PE72(.1>E:".5S67E30">64"92W>6"1EJ":2A5:8"9N">7U65X324WE6W"*$) ("J8S4268J0"H>7S38N">64"J8>HH"H5:"812E:"H32^E9E3E8N">64">WE3E8N"E6">4VSJ8E6W"85"812":62X"X>NJ"5H"82>71E6W0"32>:6E6W">64"X5:UE6W"4S:E6W"812J2"71>3326WE6W"8EC2JB"" $$$

'12"K2>6"1EW13EW1824"E6E8E>8ED2J"81>8"9:E6W"A25A32"85W2812:"JS71">J"812"*\$)(")8>N">8", 5C2".3S90">"

J8S4268JB" ' 12" * \$) (" - :>4S>82") 8S4268J"R2J2>:71". 56H2:2672"EJ"A:2A>:E6W"H5:">"DE:8S>3"2D268"E6"812"XE682:"

"

75CCS6E8N"2>71"N2>:0"H57SJE6W"56"812">X>:4"7>82W5:E2J0"65CE6>8E56">64"23EWE9E3E8N"7:E82:E>0">64"85" 756JE42:"812"A5JJE93N"5H"2J8>93EJ1E6W">"62X">X>:4"7>82W5:N"81>8"X5S34":275W6EM2"812"7568:E9S8E56J"5H" H>7S38N"C2C92:J"26W>W24"E6"4ED2:J2"H5:CJ"5H"82>71E6W">78EDE8N"81>8"C>N"658"92"7>A8S:24"9N"812"2^EJ8E6W" >X>:4JB"

6">"C58E56"4S3N"C5D240"J2756424">64"7>::E240"\$'"]T)"R()/P(K0"

', T'"812"A:5A5J24">C264C268J"85"812"'2:CJ"5H"R2H2:2672"H5:"*\$) (_J"'2>71E6W" (^72332672" TX>:4J"92"TGGR*P(K"2HH278ED2"ECC24E>823NB"

- * H"g!"C2C92:J"A:2J268">64"D58E6W0"`a"X2:2"E6"H>D5S:0"6562"5AA5J240"f">9J8>E624""
- R9 6,D<+/-!A+<@!\$/2.?3.7!&<@@3//,,-!!
- **:9 :12?**, @31!*+<7+2@-"h"G:5H2JJ5:".>:53".>CA92330".1>E:"5H"812"T7>42CE7"G:5W:>CJ".5CCE88220":2A5:824" 56"812".5CCE8822_J"A:5A5J>3J">AA:5D24">8"812"C228E6W"1234"56"*78592:"<0";=;=B"

 $!'' \# \& \& \#''' () * \# > \forall 5: " \# 54 \texttt{EHE7} > \$ \texttt{E56"G: 5A5J} > \texttt{3J"h"}) \texttt{EW6EHE7} > \texttt{68"} . 1 > \texttt{6W2J"} \$ \texttt{5"} (\texttt{EJ} \$ \texttt{E6W"G: 5W: > CJ"} = \texttt{CJ} = \texttt{CJ}$

\$

(,=!A3,>?!3.!''...3.,!B,21H3.7!2.?!),2+.3.7!L!&5++315>5@!2.?!*,?27<7S!T I %?U!

G:5H2JJ5:".>CA9233"E6DE824"G:5H2JJ5:".3>:2"O:288"85"JA2>U"85"812"A:5A5J>38"G:5H2JJ5:"O:288"65824"81>8" 812"K2A>:8C268"EJ"D2:N"2^7E824">95S8"81EJ">7>42CE7"E6E88E>8ED20"X1E71"1>J"9226"E6"A:2A>:>8E56"H5:">"356W" 8EC2B")12"1EW13EW1824"81>8"812"A:5A5J24"62X"HE234"E6"*63E62"'2>71E6W">64"/2>:6E6W"EJ"42JEW624"H5:" 24S7>85:J"X15">:2"E682:2J824"E6"9275CE6W"2HH278ED2"E6J8:S785:J">64"42JEW62:J"5H"563E62"75S:J2JB"'1(:)-3-1(T)14

81E:4"N2>:"E6" . 1E6>0">356W"XE81"812E:"5812:" #)7"E6"TAA3E24"GJN71535WN":2ZSE:2C268JB"

"